A many-valued logic intended to model silence

Osorio-Galindo Mauricio[1], Garcés-Báez Alfonso[2], and López-López Aurelio[3]
[1]osoriomauri@gmail.com
[2]alfonso.garces@correo.buap.mx
[3]allopez@inaoep.mx

[1]Universidad de las Américas - Puebla
Sta. Catarina Mártir, Cholula, Puebla, México
[2]Fac. de Cs. de la Computación. Benemérita Universidad Autónoma de Puebla.
Ciudad Universitaria, C. P. 72592. Puebla, Puebla, México.
[3]Instituto Nacional de Astrofísica, Óptica y Electrónica.
Sta. Ma. Tonantzintla, Puebla, 72840, México.

Abstract. Silence is a diverse and intangible concept that we learn to interpret within the context where it appears. Here we show that there are various knowledge areas that have studied such phenomenon. We argue that "silence" is a manifestation of intentional communication.
The seventh thesis of the Tractatus of Wittgenstein focuses on linguistic silence: "Whereof one cannot speak, thereof one must be silent".
We claim that some well known many-valued logics can be used to interpret the notion of "silence". So, we introduce a new 5-valued paraconsistent logic that we name MS. This logic is genuine and paracomplete, and has the new value that is called s attempting to model the notion of "silence".
MS is a conservative extension of FDEe, a logic proposed by Priest. If one drops the "implication" connective from MS, one obtains FDEe. If, on the other hand, one drops the ineffability value from MS one obtains a well known 4-valued logic introduced by Avron. We present some properties of this new logic.

Keywords: many-valued logic · silence · ineffability · uncertainty · inconsistency

1 Introduction

Silence is a diverse and intangible concept that we learn to interpret within the context where it appears. Here we show that there are various knowledge areas that have studied such phenomenon. We argue that "silence" is a manifestation of intentional communication.

The seventh thesis of the Tractatus is the thesis of linguistic silence."Whereof one cannot speak, thereof one must be silent" [56].

There are characterizations of the concept of silence from various branches of knowledge. For example, the communication literature emphasizes positive aspects of silence, viewing it as a critical component of social interaction [2]. Scott [49] described silence and speech as two dialectical components of effective communication.

Silence and absence are at the heart of the process of construction of meanings as an act of suppression and is necessary to look at such a process, states Achino [20]. According to Kurzon's model and taxonomy [33], the silence is defined by language and points to three types of silence [34]: psychological, interactive, and socio-cultural. For Bohnet [31], the variants for the interpretation of silence can be: anonymous and not anonymous, the latter can be with identification and face-to-face.

For Schröter and Taylor [40], deliberate silence can include: concealment, censorship, omission, evasion, lying, deception, or metalinguistic comments, other aspects of silence that are also open for exploration. For Eco [18], the intentional silence is a sign. Other examples can be found at [21, 15, 29]. Silence from the point of view of distributed systems, can be found in [28]. Works related to the logical interpretation of the consequence of omission (silence) can be found in [22, 24, 26, 25].

Silence is not only found at the limits of words, but also at the limits of numbers. For instance, when Kasner's Googol (1 followed by 100 zeros) and Googolplex (1 followed by a Googol of zeros), and when Cantor's hierarchy of transfinite numbers or Alehps are insufficient to enumerate countable infinite classes, we will be in the paradise of silence.

Is there a logic where we can handle *silence*? Some Aristotle's laws do not apply to all language whatsoever but only to the way of using language that keeps itself within these strict limits [47]. So, we better move outside classical logic.

Many-valued logics are recognized as useful in different domains. Belnap claims that a 4-valued logic is a suitable framework for computerized reasoning [32]. Avron in [5, 4, 6, 3] supports this thesis. He shows that a 4-valued logic naturally expresses true, false, inconsistent or uncertain information. Each of these concepts is represented by a particular logical value. In some cases, perhaps a wise attitude regarding inconsistent or uncertain information is to remain silent. Therefore we explore the idea of considering uncertain information as a first form of silence. We also explore the idea of considering inconsistent information as a second form of silence.

Priest argues in [43] that a 4-valued logic models quite well the four possibilities explained above, but now in the context of Buddhist meta-physics [51]. Hence, we can use 4-valued logic to represent *silence*.

Priest did a remarkable work by studying in great detail buddhist texts, from the Pali Canon to the MMK by Nāgārjuna, and being able to model their way of reasoning in terms of modern non-classical logics [43–46]. A main issue is to represent the notion of ineffability. The fifth value e is then the value of ineffable. Ineffability is our final notion of silence explored in this paper.

There is a complex but well known phenomenon that often arises when philosophy argues that there are limits to thought/language, and tries to justify this view by giving reasons as to why there are things about which one cannot think/talk—in the process appearing to give the lie to the claim. In poetry, we also find a similar situation: the need to talk about extreme situations, which somehow we can not talk [10, 36]. Currently there is much written in literature (poetry, stories) and philosophy related to apparently illogical concepts such as the paradoxical and the ineffable, we enumerate some examples next.

The theory of the two truths began twenty-five centuries ago. It started in the sixth century BCE India with Siddhārtha Gautama, who became a buddha "awakened one" because he understood: the meaning of the two truths and the reality of all the objects of knowledge is exhaustively comprised of the two truths. According to the Samādhirāja-sūtra, the theory of the two truths is a contribution made by the Buddha towards Indian philosophy. Nāgārjuna, in his Mūlamadhyamakakārikā, attributes the two truths to the Buddha as follows: "the Dharma taught by the buddhas is precisely based on the two truths: a truth of mundane conventions and a truth of the ultimate" [52]. The theory of the two truths is the heart of the Buddha's philosophy according to what the Madhyamaka philosophers indicate. The knowledge of the ultimate truth informs us of how things really are ultimately, and thus takes our minds beyond the limits of conceptual and linguistic conventions [52].

Another example is [13], where the author indicates: "My paper will focus on the ways Woolf disfigures and refigures literature through the image of ruins. Writing about the paradoxical nature of decay, Woolf captures the ineffable quality of a time that inexorably passes yet is shaped by surviving reminiscences." More evidence is the Strange Loop which is a paradoxical construction, a shift from one level of abstraction to another that somehow gives rise to a closed, eternal cycle. The author of [8] indicates that this paradoxical model is prevalent in Jorge Luis Borges's short stories and that we are able to better explore Borges's belief in literature's unique power to create spatiotemporal paradoxes. In [8], the author also analyzes how Borges creates Strange Loops in impossible linkages between distinct narrative frames and he also demonstrate how Borges composes an architectural Strange Loop.

We also can find that formerly, eastern thinkers and western mystics were keenly aware of the predicative ineffability of the ultimate. Heidegger thought and wrote about it in terms of different ideas for example "non-truth as the precondition of truth". In the Chinese side, it is found that Confucius was highly sensitive to the problem and gave his own response to it. In [57], the relation between Heidegger and analytic philosophers on this issue is analyzed, and inquire comparatively how Heidegger and Confucius recognized the limit of language, and how this awareness helped them understand the thinking role of the poetry experience.

On the other hand, the connection between the work of Martin Heidegger (1889 – 1976) and Chan/Zen Buddhism – a school of Buddhism originating in China around the 6th Century, is known. In [12], one aspect of that connection is explored, drawing on the work of the Japanese Zen philosopher Dōgen Kigen (1200 – 1253). Heidegger held that being is ineffable, and Dōgen held that ultimate reality is ineffable. More evidence is presented in [19] where one can find: "The ineffable is after all beyond words, and those who have an ineffable experience may feel so overwhelmed by the very ineffability of their experience, or emotion, that they resign themselves to saying nothing"; and "Words will always be saying too much or too little... Oh to be silent! Oh to be a painter!". Finally, we can observe that even in the context of the Christian religion, the Bible also refers to the concept of ineffable in the book of Romans, verse 8.

Despite all the interest in the subject, we can notice that little has been attempted to formalize these concepts in terms of logic. Priest is concerned with this phenomenon. According to him, Buddhist philosophy has resources to address this kind of issue much less present in Western traditions. Buddhist logicians consider that there are four possibilities: only true, only false, both true and false, and finally neither true or false. Later developments add a fifth possibility: ineffability[1]. Of course, one might be skeptical that such ideas can be done logically respectable. Priest shows how to accomplish this task with some tools from contemporary non-classical logic. His work is impeccable, but as stated earlier, we consider prudent to extend FDEe logic with an "implication" connective that at least satisfies Modus Ponens.

For the nature of this work is desirable to consider the use of paraconsistent logics [16]. In addition, recent work on these logics considers also some useful relatively new properties, namely genuineness and paracompleteness [9, 30, 41, 37, 6]. Arguments in favor of rejecting the law of non-contradiction have been supported more recently by the research done on paraconsistent logics and the applications they have encountered, in particular, in artificial intelligence. Paraconsistent logics accept inconsistencies without presenting the problem that once a contradiction has been derived, then any proposition follows as a consequence, as is the case of classical logic.

We will observe that a major issue in our logics of silence is how to define the "implication" connective. We would like to point out that adding an "implication" connective to non-classical logics is not always straightforward. Corcoran has distinguished twelve uses of the term "implies" [14]. There is nothing to prevent "implies" to be interpreted in any conventional way in order to convey any

[1] As far as the authors know, buddhist texts never talk explicitly about five possibilities, as they actually mention four cases. However, Priest shows that buddhist narratives assume this sort of incommensurable fifth, see [43–46].

pragmatical use. We could name it differently, but since we expect it to obey the Modus Ponens rule of inference, we decided to call it "implication".

Important remark: This paper does not try to present a particular position about "silence" as, for instance, the claim given in the Tractatus mentioned before. The point of the paper is that the notion of "silence" is important to be studied and modeled using logic.

Our paper is structured as follows. In section 2, we summarize some definitions, logics and semantics necessary to understand this work. Then in section 3, we present our results. Namely, we introduce the logic MC and some of its properties. Finally, in section 4 we present some conclusions.

2 Background

Classical logic is a logic that obeys the law of excluded middle (or the principle of excluded middle) which states that for any proposition, either that proposition is true (t) or its negation is true, where in this case we say that the original proposition is false (f). The law is also known as the law (or principle) of the excluded third, in Latin *principium tertii exclusi*. Another Latin designation for this law is *tertium non datur*: "no third (possibility) is given".

In this section, we define a logic from the semantical point of view, particularly via muti-valued systems, so we present one axiomatic formal system for logic *bl*, provided by Avron in [7]. Then, we review the system HBL, a formal axiomatic theory for *bl* [5]; and finally we present a summary of some material from [44] that we need to borrow for the definition of our contribution.

2.1 Multi-valued logics

A way to define a logic is by means of truth values and interpretations. Multi-valued systems generalize the idea of using the truth tables, employed to determine the validity of formulas in classical logic. It has been suggested that multi-valued systems should not count as logics; on the other hand pioneers such as Łukasiewicz considered such multi-valued systems as alternatives to the classical framework. As other authors, we prefer to give to multi-valued systems the benefit of doubt about their status as logics. The core of a multi-valued system is its *domain* of values D, where some of such values are special and identified as *designated*. Connectives (e.g. $\wedge$, $\vee$, $\rightarrow$, $\neg$) are then introduced as operators over D according to the particular definition of the logic. An *interpretation* is a function $I\colon \mathcal{L} \rightarrow D$ that maps atoms to elements in the domain. The application of I is then extended to arbitrary formulas by mapping first the atoms to values in D, and then evaluating the resulting expression in terms of the connectives of the logic. A formula is said to be a *tautology* if, for every possible interpretation, the formula evaluates to a designated value. Given a set of formulas Γ, and a formula α, the notation $\Gamma \models \alpha$ means that any interpretation that models all formulas of Γ also models α. The most simple example of a multi-valued logic is classical logic where: $D = \{0, 1\}$, 1 is the unique designated value, and connectives are defined through the usual basic truth tables. From now on, we refer to all multi-valued systems as multi-valued logics.

Not all multi-valued logics must have the four connectives mentioned before, in fact classical logic can be defined in terms of two of those connectives $\neg$, $\wedge$ (primitive connectives), and the other two (non-primitive) can be defined in terms of $\neg$, $\wedge$. In case of a logic having the implication connective, it is desirable that it preserves tautologies, in the sense that if $\alpha, \alpha \rightarrow \beta$ are tautologies, then β is also a tautology. This restriction enforces the validity of Modus Ponens in the logic. The Deduction theorem (in the context of model theory) is another suitable property, namely: $\Gamma, \beta \models \alpha$ iff $\Gamma \models \beta \rightarrow \alpha$.

Objects 0, 1, 2, 3 and 4 are part of the semantics of logics studied in this paper and were chosen only for convenience, they do not correspond to the natural numbers.

A logic satisfies the principle of explosion (EFQ) if $\alpha, \neg\alpha \models \beta$. A logic is paraconsistent if it rejects the principle of explosion. A logic satisfies the principle of non-contradiction (PNC) if $\models \neg(\alpha \wedge \neg\alpha)$. A logic is genuine if it rejects the principle of non-contradiction. A logic satisfies the law of excluded middle if $\models \alpha \vee \neg\alpha$. A logic is paracomplete if it rejects the law of excluded middle.

A 3-valued logic is any of several many-valued logic systems with three truth values indicating true, false and some indeterminate third value. We could say that this new third valued is neither false nor true (n). The conceptual construction of 3-valued logics and its basic ideas were initially created by Jan Łukasiewicz and C. I. Lewis. The core of all our 3-valued logics is based on the following assumptions.

We have 3 values: $\{0, 1, 2\}$. At least we have one designated value which is 2. It defines a lattice where $0 < 1$ and $1 < 2$. We use $0, 1$ and 2 to represent False, Unknown and True respectively.

The connective $\vee$ is the Lub, while $\wedge$ is the Glb. Regarding negation ($\neg$), we have $\neg 0 = 2$, $\neg 1 = 1$ and $\neg 2 = 0$. Table 1 presents the truth tables of connectives $\vee$, $\wedge$, and $\neg$.

It is important to remark that most well known 3-valued logics in the literature share these assumptions.

Kleene and Priest Logic of Paradox (LP). In Kleene strong 3-valued logic, the third value (1), can be thought of as neither true (2) nor false (0). Notice that also Łukasiewicz 3-valued logic corresponds to Kleene logic on these operators and designator values.

On the other hand, in Priest 3-valued logic (LP), this third value (1) is thought of as both true (2) and false (0). The difference lies on the definition of tautologies. Where Kleene logic's only designated truth value is T (2), Priest logic's designated truth values are both T (2) and U (1). Notice also that PAC 3-valued logic corresponds to LP on these operators and designator values [7].

LP accepts that we can gave both true and false propositions. We can use the third value (1) to represent this fact. A major example in Mathematics comes from set theory. It is known from Russell's paradox that the first-order axiom-

∨	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

∧	0	1	2
0	0	0	0
1	0	1	1
2	0	1	2

x	$\neg x$
0	2
1	1
2	0

Table 1. Truth tables of connectives ∨, ∧, and ¬.

$\to_K$	0	1	2
0	2	2	2
1	2	2	2
2	0	1	2

$\to_P$	0	1	2
0	2	2	2
1	0	1	2
2	0	1	2

Table 2. Truth tables of connectives $\to_K$ and $\to_P$.

atization of the naive set theory of Cantor and Frege is inconsistent in classical logic. More precisely, some "peculiar" sets lead to triviality if the underlying logic is classical. The most popular of those is the so-called Russell set, $R = \{x|x \notin x\}$, that by the law of excluded middle, one immediately gets an apparent contradiction. To handle such a contradictory set, a possible idea is to take the Russell set just for granted, namely as a set that belongs and does not belong to itself [35]. It is interesting to note that non-well-founded sets (also called hypersets), which are no more indispensable for the foundations of Mathematics, have subsequently found useful applications in modeling circular phenomena, particularly in Computer Science [1].

In both logics, material implication can be defined as: $X \to Y = (\neg X) \vee Y$. There are many and well known arguments against reading material implication as any kind of implication [48, 14]. In this paper, however, we define the implication operator as sugested by Avron [7], namely:

$X \to Y = 2$ when X is not designated.

$X \to Y = Y$ when X is designated.

Since 1 is designated value in logic LP and is not a designated value in Kleene logic, the table for the implication operator is different in both logics. We denote with $\to_K$ the implication operator that we are assigning to Kleene logic. We denote with $\to_P$ the implication operator that we are assigning to Priest logic. As Avron points out, this definition of the implication operator satisfies suitable structural properties [7]. Table 2 shows both tables.

Logic $BL_{\supset}$. This logic is a 4-valued logic with truth values in the domain $D = \{0, 1, 2, 3\}$ where 2 and 3 are the designated values [5]. The connectives ∧ and ∨, as usually, correspond to the *greatest lower bound (Glb)* and the *least upper bound (Lub)*, respectively. The connectives → and ¬ are defined as:

$\alpha \to \beta = 3$ if α is not designated,

$\alpha \to \beta = \beta$ if α is designated;

$\neg(0) = 3$, $\neg(3) = 0$, $\neg(\alpha) = \alpha$ if $\alpha = 1$ or $\alpha = 2$.

The logic $BL_{\supset}$ is induced by the partial order $0 < 1$, $0 < 2$, $1 < 3$, $2 < 3$. Note that if we consider only the values 0, 1 and 3, we obtain the Kleene's logic while if we take the values 0, 2 and 3 we have the PAC logic. The Kleene and PAC logics [7] defined by 3-truth values and which have been studied in detail are sublogics of $BL_{\supset}$.

As A. Avron mentions in [5], $BL_{\supset}$ is *interlaced* [2] and hence satisfies $1 \wedge 2 = 0$ and $1 \vee 2 = 3$.

2.2 The system HBL

Let us consider HBL, a formal axiomatic theory for $BL_{\supset}$ [5] formed by the primitive logical connectives: $\neg, \rightarrow, \wedge$ and $\vee$. We also consider one logical connective defined in terms of the primitive ones:

$$\alpha \leftrightarrow \beta := (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$$

the well-formed formulas are constructed as usual, the axiom schemas are:

I1 $\alpha \rightarrow (\beta \rightarrow \alpha)$
I2 $(\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma))$
I3 $((\alpha \rightarrow \beta) \rightarrow \alpha) \rightarrow \alpha$
C1 $(\alpha \wedge \beta) \rightarrow \alpha$
C2 $(\alpha \wedge \beta) \rightarrow \beta$
C3 $\alpha \rightarrow (\beta \rightarrow (\alpha \wedge \beta))$
D1 $\alpha \rightarrow (\alpha \vee \beta)$
D2 $\beta \rightarrow (\alpha \vee \beta)$
D3 $(\alpha \rightarrow \gamma) \rightarrow ((\beta \rightarrow \gamma) \rightarrow (\alpha \vee \beta \rightarrow \gamma))$
N1 $\neg(\alpha \vee \beta) \leftrightarrow \neg\alpha \wedge \neg\beta$
N2 $\neg(\alpha \wedge \beta) \leftrightarrow \neg\alpha \vee \neg\beta$
N3 $\neg\neg\alpha \leftrightarrow \alpha$
N4 $\neg(\alpha \rightarrow \beta) \leftrightarrow \alpha \wedge \neg\beta$

and as the only inference rule: Modus Ponens

$$\frac{\alpha \quad \alpha \rightarrow \beta}{\beta}$$

Logic $BL_{\supset}$ is sound and complete with respect to this axiomatization.

$\Gamma \models_X \alpha$ means that α can be proved in logic X from formulas in Γ.

Theorem 1. *[5][Soundness and Completeness]*

$$\Gamma \vdash_{BL_{\supset}} \alpha \ if \ and \ only \ if \ \Gamma \models_{BL_{\supset}} \alpha.$$

2.3 First Degree Entailment system

This subsection is a summary of some material from [44] that we need to refer for the definition of our logics.

First Degree Entailment (FDE) is a system of logic defined by Priest that can be set up in many ways, but one of these is as a 4-valued logic whose values are t (true only), f (false only), b (both), and n (neither). Negation maps t to f, vice versa, n to itself, and b to itself. Conjunction is Glb, and disjunction is Lub. The set of designated values D is $\{b, t\}$ The four corners of truth and the FDE logic seem like a correct match. From now on, we will use the four values 0,

[2] This means that each one of $\wedge$, and $\vee$ is monotonic with respect to both $\leq_t$ and $\leq_k$ [5]

1, 2, 3 that correspond to f, n, b, t respectively; this in order to make notation uniform in terms of previously introduced logics.

FDE can be characterised by the following sound and complete rule system, where a double line indicates a two-way rule, and overlining indicates discharging an assumption[3]:

$$\frac{A, B}{A \wedge B} \qquad \frac{A \wedge B}{A(B)} \qquad \frac{A \vee B \qquad \overline{A} \ldots C \qquad \overline{B} \ldots C}{C}$$

$$\frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg\neg A}{A}$$

Now we move to FDEe, a 5-valued logic that incorporates the notion of ineffability. According to Priest, technically, the obvious thought is to add a new value (which in [44] appears as e), 4, to our existing four $\{0, 1, 2, 3\}$, expressing this new status.

Since 4 is the status of claims such that neither they nor their negations should be accepted, it should obviously not be designated. Thus, we still have that same designated values. Priest addresses the following major question: How are the connectives going to behave with respect to 4?

Both 4 and 1 are the values of things that are, in some sense, neither true nor false, but they need to behave differently if the two are to represent distinct alternatives. The simplest suggestion is to take 4 to be such that whenever any input has the value 4, so does the output: i.e. 4-in/4-out.

The logic that results by modifying FDE in this way is obviously a sub-logic of it. It is a proper sub-logic. It is not difficult to check that all the rules of FDE are designation-preserving except the rule for disjunction-introduction, which is not, as an obvious counter-model shows. However, replace this with the rules:

$$\frac{\varphi(A) \quad C}{A \vee C} \qquad \frac{\varphi(A) \quad C}{\neg A \vee C} \qquad \frac{\varphi(A) \quad \psi(B) \quad C}{(A \wedge B) \vee C}$$

where $\varphi(A)$ and $\psi(B)$ are any sentences containing the atoms that occur in A and B respectively. For example, for rule $\frac{\varphi(A) \ C}{A \vee C}$ if A is the atom a and $\varphi(A)$ is $a \wedge b$ then some possible instances of this rule are: $\frac{a \wedge b \ C}{a \vee C}$ or $\frac{a \ C}{a \vee C}$. Call these the φ *Rules*, and call this system FDE_φ. FDE_φ is sound and complete with respect to the semantics [46].

3 Construction of a new 5-valued logic

Going beyond four values (a kind of final frontier) is to accept ineffable propositions and hence accepting five logical values. It is worth to challenge modern

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[3] The paper [44] downloaded from the Priest's home page has a typo. It says: $\frac{\neg(A \vee B)}{\neg A \vee \neg B}$ instead of: $\frac{\neg(A \vee B)}{\neg A \wedge \neg B}$.

$\vee$	0	1	2	3	4
0	0	1	2	3	4
1	1	1	2	3	4
2	2	2	2	3	4
3	3	3	3	3	4
4	4	4	4	4	4

$\rightarrow$	0	1	2	3	4
0	3	3	3	3	?
1	3	3	3	3	?
2	0	1	2	3	?
3	0	1	2	3	?
4	?	?	?	?	?

x	$\neg x$
0	3
1	1
2	2
3	0
4	4

Table 3. Truth tables of connectives $\vee$, $\rightarrow$ and $\neg$.

logic to represent the above 5 notions. Priest already did the job. However he left the "implication" connective out of his approach. We present an alternative to include such connective in a logic that he proposed. We now present the construction of our logic in two steps.

The first step defines the core of our logic as follows. We have 5 values: $\{0, 1, 2, 3, 4\}$. Make sure that our logic agrees with $BL_{\supset}$, in the $\{0, 1, 2, 3\}$ fragment. Note also that FDEe also uses $\{0, 1, 2, 3, 4\}$. The designated values are $\{2, 3\}$ as FDEe. $\{0, 1, 2, 3\}$ defines a lattice where $0 < 1$, $0 < 2$, $1 < 3$, $2 < 3$. The connectives $\vee$, $\wedge$ and $\neg$ should agree with FDEe. Hence, the connective $\vee$ is the Lub, while $\wedge$ is the Glb. Since 4 is interpreted as ineffable then $X\ op\ 4 = 4\ op\ X = 4$, where $X \in \{0, 1, 2, 3, 4\}$. With respect to negation ($\neg$), we have $\neg 0 = 3$, $\neg 3 = 0$, $\neg 1 = 1$, $\neg 2 = 2$, $\neg 4 = 4$. Implication is as defined by Avron for the subdomain $\{0, 1, 2, 3\}$, namely: $X \rightarrow Y = 3$ when X is not designated, $X \neq 4$, $Y \neq 4$. While $X \rightarrow Y = Y$ when X is designated, $Y \neq 4$. The next two expressions are yet undefined $4 \rightarrow X$ and $X \rightarrow 4$ for X in $\{0, 1, 2, 3, 4\}$. Table 3 presents the truth tables of connectives $\vee$, $\rightarrow$ and $\neg$.

Notice that the sublogic defined in the subdomain $\{0, 1, 2, 3\}$ corresponds exactly to $BL_{\supset}$ logic. Also the sublogic in the domain $\{0, 1, 2, 3, 4\}$ but eliminating the implication connective corresponds exactly to FDEe.

Now we move to our second step in the construction of our logic. The idea is the ensure that our logic satisfies a number of suitable properties that satisfy both FDEe and $BL_{\supset}$ logics.

3.1 Our proposed MS logic

This logic is kind of pragmatic and the connective "implication" is a kind of metalinguistic connective [14]. We name this logic as MS. This logic is defined to complies with the tautologies **I1-I3**, **C1-C3**, **D3**, **N1-N4** (see section 2.2). The connective $\rightarrow$ is defined according to the truth table given in Table 4.

Theorem 2. *MS is a paraconsistent, genuine and paracomplete logic.*

Proof (sketch). Directly using truth tables.

We can observe that MS satisfies many well known tautologies, as $X \rightarrow X$ and De Morgan laws, the standard two implication rules for positive logic among some of them. Hence, we have the following theorem.

$\to_{FiveASP3}$	0	1	2	3	4
0	3	3	3	3	3
1	3	3	3	3	3
2	0	1	2	3	4
3	0	1	2	3	4
4	3	3	3	3	3

Table 4. Truth table of connective $\to$ in logic MS.

Theorem 3. *All axiom shemas of HBL are tautologies in MS logic but D_1 and D_2.*

Proof (sketch). The proof is direct by checking the truth tables of MS.

Theorem 4. *MS satisfies:*

1. *Modus Ponens, the Deduction theorem and Hypothetical Sylogism.*
2. *All inference rules of FDEe.*

Proof (sketch). (case 1) They are proven by contradiction using truth tables. (case 2) They are proven by construction, since the three logics behave as logic FDEe regarding the connectives $\wedge$, $\vee$ and $\neg$, and that logic satisfies such inference rules.

Note that MS is different from the 3 logics proposed in [39]. We consider this logic potentially useful to model different forms of *silence*. Value (1), can be thought of as neither true (3) nor false (0). Hence it interprets silence as uncertainty. In some cases, a wise attitude regarding uncertain information is to remain silent. Value (2), can be thought as both true (3) and false (0). Hence it interprets silence as inconsistency. A wise attitude regarding inconsistent information, in certain situations, is to remain silent. Value (4), can be thought as ineffability. Hence it interprets silence directly as ineffability. Again, in some cases, a wise attitude regarding ineffability is to remain silent.

4 Conclusions and future work

4.1 Conclusions

Silence is a concept that has been studied from diverse perspectives, but remains elusive of a formal approach. Here, a new formulation is offered.

The proposed 5-valued paraconsistent logic allows to handle at least three situation where silence can occur. This expands the possibilities of reasoning with such logic. For instance, a possible application can be in the legal domain where often those involved in a case decide to remain silent.

Being consistent with Wittgenstein, beyond the artifices to name the cardinality of the infinitely large and the infinitely small is the paradise of silence.

4.2 Future work

We would like to explore the use of the proposed 5-valued logic on a test set, to evaluate its utility. One possibility is to apply it on puzzles, as in [22, 26].

Other possible future work could be related to language applications, for example, in dialogues or when we cannot or do not want to make a judgment that is equivalent to holding that a statement is true (yes) or false (no). We could prefer a third option as to *shut up* with the possibility that whoever *resorts to silence* has the intention of sending, veiled, an affirmation or a denial depending on the context. So a simplified 3-valued logic could also be formalized, resembling Kleene's 3-Valued Logic.

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References

1. Samson Abramsky. A Cook's tour of the finitary non-well-founded sets. Department of Computing, Imperial College, London, 1988.
2. Haig Khatchadourian. *How to Do Things with Silence. Walter de Gruyter GmbH & Co KG, 2015.*
3. Ofer Arieli and Arnon Avron. Logical bilattices and inconsistent data. In *Proceedings of the Ninth Annual Symposium on Logic in Computer Science (LICS '94), Paris, France, July 4-7, 1994*, pages 468–476, 1994.
4. Ofer Arieli and Arnon Avron. Four-valued diagnoses for stratified knowledge-bases. In *Computer Science Logic, 10th International Workshop, CSL '96, Annual Conference of the EACSL, Utrecht, The Netherlands, September 21-27, 1996, Selected Papers*, pages 1–17, 1996.
5. Ofer Arieli and Arnon Avron. Reasoning with logical bilattices. *Journal of Logic, Language and Information*, 5(1):25–63, 1996.
6. Ofer Arieli and Arnon Avron. Four-valued paradefinite logics. *Studia Logica*, 105(6):1, 2017.
7. Arnon Avron. Natural 3-valued logics–characterization and proof theory. *Journal of Symbolic Logic*, 56(1):3–4, 03 1991.
8. Jessica Erin Beebe. The Strange Loop: Paradoxical Hierarchies in Borges's Fictions. *Honors Projects*, 11, 2014. *URL* =< https://digitalcommons.spu.edu/honorsprojects/11 >.
9. Jean-Yves Beziau. Two genuine 3-valued paraconsistent logics. In *Towards Paraconsistent Engineering*, pages 35–47. Springer, 2016.
10. E. Black. Mouthlessness and ineffability in world war i poetry and the waste land. *War, Literature and the Arts: An International Journal of the Humanities*, 25:1–17, 2013.
11. Norman O. Brown. *Apocalypse and/or Metamorphosis*. University of California Press, 1991.

12. Filippo Casati and Graham Priest. Heidegger and Dōgen on the Ineffable. *The Significance of Indeterminacy: Perspectives from Asian and Continental Philosophy*. Eds. Roger H. Scott and Gregory S. Moss. Routledge, 2019, pp. 279-308.
13. Adèle Cassigneul. Virginia WoolfÂ´s Ruined House, a Literary Complex. Études britanniques contemporaines [On-line], 43, 2012, $URL =<$ http://journals.openedition.org/ebc/1315>.
14. John Corcoran. Meanings of implication. *Diálogos. Revista de Filosofía de la Universidad de Puerto Rico*, 9(24):59–76, 1973.
15. John Chatwin, Penny Bee, Gary Macfarlane, Karina Lovell and others. Observations on silence in telephone delivered cognitive behavioural therapy (T-CBT). *Journal of Linguistics and Professional Practice*, 1(11). Equinox Publishing. 2014.
16. Newton C. A. da Costa, Décio Krause, and Otávio Bueno. *Paraconsistent Logics and Paraconsistency*, pages 791–911. Elsevier, 2007.
17. Didier Dubois and Henri Prade. *Possibility Theory - An Approach to Computerized Processing of Uncertainty*. Springer, 1988.
18. Umberto Eco. *El signo*. Labor Bogotá, 1994.
19. Sophie Fenella Robins. Say Nothing. Say Anything. Do Something: Expressing the Ineffable in Performance Poetry. *Liminalities: A Journal of Performance Studies*, 10(3/4):1–27, 2014.
20. Maria-Luisa Achino-Loeb. Introduction: Silence as the currency of power *Silence: The currency of power*, pages 1–22, 2006.
21. Joanna Garbutt The Use of No Comment by Suspects in Police Interviews. *Exploring Silence and Absence in Discourse*, pages 329–357. Springer. 2018.
22. Alfonso Garcés-Báez and Aurelio López-López. Reasoning in the Presence of Silence in Testimonies: A Logical Approach *Intelligent Computing*, pages 952–966, Springer, 2021.
23. Alfonso Garcés-Báez and Aurelio López-López. A Logical Interpretation of Silence. *Computación y Sistemas* 24(2). 2020.
24. Alfonso Garcés-Báez and Aurelio López-López. Towards a Semantic of Intentional Silence in Omissive Implicature. *Digitale Welt*, pages 67–73. Springer. 2020.
25. Alfonso Garcés-Báez and Aurelio López-López. A semantics of intentional silence in omissive implicature. *Journal of Intelligent & Fuzzy Systems*, pages 2115–2126. IOS Press. 2020.
26. Alfonso Garcés-Báez and Aurelio López-López. First Approach to Semantics of Silence in Testimonies. *International Conference of the Italian Association for Artificial Intelligence, LNAI 11946*, pages 73–86. Springer. 2019.
27. Brendan S. Gillon. An early budhist text on logic: Fang Bian Xin Lun. *Argumentation*, 1(22):15–25, 2008.
28. Guy Goren and Yoram Moses. Silence. *Proceedings of the 2018 ACM Symposium on Principles of Distributed Computing*, pages 285–294. 2018.
29. Heidi M Levitt. Sounds of silence in psychotherapy: The categorization of clients' pauses. *Psychotherapy Research*, pages 295–309. Taylor & Francis. 2001.
30. A. Hernández-Tello, J. Arrazola Ramírez, and M. Osorio Galindo. Mouthlessness and ineffability in world war i poetry and the waste land. *Log. Univers.*, 11(4):507â€“–524, 2017.
31. Iris Bohnet and Bruno S Frey. The sound of silence in prisoner's dilemma and dictator games. *Journal of economic behavior & organization*, pages 43–57. Elsevier. 1999.
32. Dunn J.M. and Epstein G., editors. *Modern Uses of Multiple-Valued Logic*, volume 2 of *Episteme (A Series in the Foundational, Methodological, Philosophical,*

Psychological,Sociological, and Political Aspects of the Sciences, Pure and Applied), chapter A Useful Four-Valued Logic. Springer, Dordrecht, 1977.
33. Dennis Kurzon. The right of silence: A socio-pragmatic model of interpretation. *Journal of Pragmatics*, pages 55–69. Elsevier. 1995.
34. Dennis Kurzon. Towards a typology of silence. *Journal of Pragmatics*, pages 1673–1688. Elsevier. 2007.
35. Thierry Libert, Models for a paraconsistent set theory. *Journal of Applied Logic*,3(1):15–41,2005.
36. Lowry Nelson. The rhetoric of ineffability: Toward a definition of mystical poetry. *Comparative Literature*, 8(4):323–336, 1956.
37. M. Osorio, J.L. Carballido, and C. Zepeda. SP3B as an extension of C_1. *South American Journal of Logic*, 4(1):1–27, 2018.
38. Mauricio Osorio and Juan Carlos Nieves. Pstable semantics for possibilistic logic programs. In *MICAI 2007: Advances in Artificial Intelligence, 6th Mexican International Conference on Artificial Intelligence, Aguascalientes, Mexico, November 4-10, 2007, Proceedings*, pages 294–304, 2007.
39. Mauricio Osorio and Claudia Zepeda. Three New Genuine Five-valued Logics Intended to Model Non-trivial Concepts. Electronic Notes in Theoretical Computer Science. Vol 354. pages 157-170, 2020.
40. Melani Schröter, and Charlotte Taylor. *Exploring silence and absence in discourse: empirical approaches* Springer, 2017.
41. Y. Petrukhin. Generalized correspondence analysis for three-valued logics. *Log. Univers.*, 12(3):423–460, 2018.
42. G Priest, K Tanaka, Y Deguchi, and J Garfield, editors. *The Moon Points Back*, chapter Nāgārjuna's logic. Oxford University Press, 2015.
43. Graham Priest. The logic of the catuskoti. *Comparative Philosophy*, 1(2):24–54, 2010.
44. Graham Priest. None of the above: The catuskoti in indian buddhist logic. In *New Directions in Paraconsistent Logic*, pages 517–527. Springer, 2015.
45. Graham Priest. Speaking of the ineffable, east and west. *European Journal of Analytic Philosophy*, 11(2):6–20, 2015.
46. Graham Priest. *The Fifth Corner of Four. An Essay on Buddhist Metaphysics and the Catuskoti.* Oxford University Press, 2019.
47. Graham Priest. To be and not to be-that is the answer. On Aristotle on the law of non-contradiction. *Philosophiegeschichte und Logische Analyse*, 91–130 1998.
48. Graham Priest. The logic of paradox. *Journal of Philosiphical logic*, 8:219–241, 1979.
49. Robert L Scott. Dialectical tensions of speaking and silence. *Quarterly Journal of Speech*, 79(1):1–18, Routledge, 1993.
50. Jesús Silva-Herzog Márquez. Andar y Ver. El blog de Jesús Silva-Herzog Márquez. La caída de Octavio Paz. Available from: *URL* =<https://www.andaryver.mx/arte/poesia/la-caida/>.
51. Eric Swanson and Mingyur Rinpoche Yongey. *Joyful Wisdom: Embracing Change and Finding Freedom.* Three Rivers Press, 2010.
52. Thakchoe, Sonam. *The Theory of Two Truths in India.* The Stanford Encyclopedia of Philosophy (Spring 2017 Edition), Edward N. Zalta (ed.), *URL* =<https://plato.stanford.edu/archives/spr2017/entries/twotruths-india/>.
53. George Steiner. *Lenguaje y silencio: ensayos sobre la literatura, el lenguaje y lo inhumano.* Editorial Gedisa, 2020.
54. Luis Villoro. *La significación del silencio y otros ensayos.* Fondo de Cultura Económica, 2016.

55. Wikisource contributors. Tractatus Logico-Philosophicus/Introduction [Internet]. Wikisource; 2016 May 10, 21:15 UTC [cited 2019 Nov 5]. Available from: URL =<https://en.wikisource.org/w/index.php?title=Tractatus_Logico-Philosophicus/Introduction&oldid=6228399>.
56. Wittgenstein. *Tratado Lógico-Filosófico*. Gredos, 2016.
57. Xianglong Zhang. From the Ineffable to the Poetic: Heidegger and Confucius on Poetry-Expression of Language. *In From the Ineffable to the Poetic: Heidegger and Confucius on Poetry-Expression of Language*, Leiden, 2018. The Netherlands: Brill.